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Title of article: Using Children's Work in a Content Course to Support the Development of Preservice Elementary Teachers' Mathematical Knowledge for Teaching Fractions

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Abstract: This article describes the creation and implementation of a module in a mathematics for teaching course that was centered around children's written work and verbal explanations. The goal of the module was to support the development of preservice elementary teachers' mathematical knowledge for teaching fractions (MKTF).

Key Words:

Teacher educators are constantly searching for ways to prepare preservice teachers to enter the field of education. One such way that has gained prominence in the last 20 years is through the utilization of children's work. Research suggests that using children's work helps preservice teachers gain insight into their own beliefs about what it means to know and do mathematics (Timmerman 2004) and acquire some elements of the knowledge needed for teaching (Tirosh 2000). Hill, Schilling, and Ball (2004) described this type of knowledge as *mathematical knowledge for teaching* (MKT). MKT, they argue, is different than the mathematical knowledge needed by mathematicians and encompasses, among other things, understanding the concepts underlying procedures, performing error analysis to determine where a student's mistake lies, and choosing effective representations to model concepts.

Encouraged by the seemingly many benefits of using children's work in other teacher education courses, we were interested in incorporating it in to a mathematics for teaching course. Specifically, we aimed to support the development of preservice elementary teachers' mathematical knowledge for teaching fractions (MKTF). To accomplish this, we created a 3-day module from four 5th-graders' written work and verbal explanations which was implemented in one section of a mathematics for teaching course during the spring of 2006. A total of 23 female preservice teachers, all of who were juniors or seniors, participated in the activities. In this article, we briefly discuss the development and implementation of the module, share some observations and reflections made during the pilot implementation, and detail some of the questions that arose for us.

Module Development and Implementation

In December 2005, the facilitators of a multi-school, grade 3-6 teacher study group within the Center for the Mathematics Education of Latinos/as (CEMELA)[1] invited four 5th-grade students to share their thinking about how to compare fractions. The students, two Hispanic females and two African-American males, were asked to participate because of their varying levels of fractional knowledge.

The students were asked to complete the worksheet "Comparing Fractions," a part of the 4th-grade *Investigations in Number, Data, and Space*® curriculum. Each of the students was then videotaped sharing his/her thinking about one of the problems s/he completed. After the students left, the teachers of the study group engaged in a rich discussion about the students' thinking of fractions, with the teachers mainly focusing on the fragile understandings the students appeared to have.

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Upon reviewing the video clips, we, along with other members of CEMELA, decided that the children's written work and explanations should be developed into a module for a university mathematics course for elementary education majors. The course, which examines K-8 mathematics from a more advanced perspective, has a history of examining children's explanations, specifically around the concepts of multiplication and division. Since this part of the course is generally met with a great deal of enthusiasm, we were interested in seeing how preservice teachers would engage with children's thinking when the content is fractions, an area that many preservice teachers generally find more difficult (Ball 1990).

During spring 2006, we (the authors and the course instructor[2]) developed four units, one for each child (whom we call Student Y, Student D, Student A, and Student T). Each unit was comprised of two sections: one which focused on the children's written work (labeled Section I) and another which focused on the children's verbal explanations of their written work (labeled Section II). Figure 1 shows an example of a unit that was developed.

Before implementing the activities that specifically focused on the children's work, the course instructor had the preservice teachers take a prior knowledge assessment and then held small group and whole-class discussions

[2] The authors wish to thank Dr. Erin McNicholas for her insight and time in developing and implementing the module.

about their responses. The assessment we developed consisted of six questions related to fractional topics such as representation, comparison, the whole, and equivalency. After these discussions, the preservice teachers completed Section I of each unit, the analysis of each of the children's written work, followed by Section II, the investigation into the children's verbal explanations of their written work. The format of having the preservice teachers look at the written work independent of the verbal explanations was chosen deliberately. We hypothesized that the preservice teachers would be more engaged in listening to the children's explanations once they spent time analyzing the children's written work.

While observing the preservice teachers engage in classroom discussions and then looking through their completed work, we noted how the preservice teachers were developing MKTF. Below we discuss two items that were of particular interest to us. The first is the preservice teachers' understanding about the representation of fractions, both of their own and that of their future students. The second is in regard to the preservice teachers' mathematical understanding of fractions through their discussions of the children's work.

Representations of fractions

One area of focus throughout the module was the representation of fractions. Hill, Schilling, and Ball (2004) argue that choosing effective representations to model concepts is part of possessing MKT. To assess the preservice teachers' MKT with regard to their representation of fractions, they were asked on the prior assessment to represent 5/6 in two different ways. In the discussion that followed, one preservice teacher shared two different area models - a hexagon divided into six equal pieces with five pieces shaded and a rectangle divided into six equal pieces with five pieces shaded. Following this, another preservice teacher (PT 1) offered her representation of 5/6, shown in figure 2. After PT 1 drew her representation, another member of the class (PT 2) commented that the number line drawn by PT 1 should be divided in to equal-sized segments in order to accurately represent 5/6.

> PT 2: "But is that another representation of it [5/6] or is it just another way to write it? Because I don't think that [the number line representation of the fraction] shows me what 5/6 is. It just puts it between 0 and 1."
>
> PT 1: "Well, it's like, this [pointing to the two area models on the board] is more like concrete, you know you can touch it. This is just like a number line. It's showing that 5/6 is less than one, greater than zero, but it's closer to one."

PT 2: “Yeah, but you still didn’t partition them off so I could see … [PT 1: “Oh, well…”] … so I could see.”

PT 1: “I could do like 1/6, 2/6, 3/6, 4/6 [drawing tick marks on the number line]… I could do it that way.”

PT 2: “That way it’s easier for me to see. It’s not that, it’s just sort …”

PT 1: “But then it’s just like that [pointing to the two area models].”

PT 3: “But at least we can see the progression. It’d be good for students to see …”

PT 2’s argument appears to be that the number line should show parts of the whole, with the number line partitioned in to sixths. However, it seems that PT 1 had a different idea than PT 2 of the meaning of 5/6, where PT 1 viewed the fraction as a number within a larger set of numbers. The problem for PT 2 was how to comprehend what PT 1 was drawing in order for her own representation to make sense. What PT 2 needed to see was the number line showing parts of a whole, evidenced by her repeated use of phrases such as “so I could see…” and “it doesn’t show me.” This difficulty in coordinating two different interpretations is not uncommon and is something with which elementary children may struggle.

In fact, at the end of the exchange, a third preservice teacher, PT 3, joined the conversation in which she switched the focus from the preservice teachers’ own understanding of 5/6 to what representation might aid elementary students. This focus on elementary students continued after this discussion. After observing their in-class interactions and looking through their completed work, we noted that many of the preservice teachers commented that it would be important to expose children to different representations in order to help them make sense of fractional concepts. For example, one preservice teacher commented that she would use "realistic things so [a] child could relate," while another cautioned using only area models saying, "I believe that [the] area model, while visually appealing, could confuse the child …"

Another example of how the preservice teachers worked to make sense of their own mathematical understandings about representations occurred when three of the preservice teachers discussed Student T's work. Student T had concluded that 2/3 and 5/6 are equivalent by focusing on the fact that each fraction had one leftover piece (see figure 3 for a copy of Student T's work and a transcript of his explanation).

PT 1: "His whole equal sign thing … I don't know."

PT 2: "I don't know what it means."

PT 3: "Maybe he was thinking of comparing the equivalent fraction with [a] denominator like 6? It still would be wrong [PT 1: Yeah] so I don't know."

. . .

PT 2: "The way he has written it, he needs a better model. The divisions aren't the same [PT 1 & PT 3: Yeah]."

PT 3: "He is having trouble with which leftover piece is bigger."

PT 3: "Again though, the rectangles are the same size and they both show that one box is left, so…"

PT 2: "You know, I see that, he thinks he needs to add that piece but he doesn't and doesn't shade it in."

PT 1: "You know what? With those numbers of pieces left, his equal size, 'cuz like there is one piece left in both …"

In this discussion, the three preservice teachers reflected on their own understanding of fractions to make sense of how someone could conclude that 2/3 and 5/6 were equivalent. In the beginning, they struggled to understand what the student was doing, which is evident in their repeated use of phrases such as "I don't know." However, toward the end of the exchange, the preservice teachers connected their own fractional knowledge with the representations drawn, seeing that because there was one piece left in each drawing, it was possible that Student T believed they were equivalent fractions. We posit that this reflection and use of mathematical knowledge was possible because of the module activities.

The examples that we present showcase that during the children's thinking activities the preservice teachers were able to reflect on and use their knowledge of fractional representations as well as consider what representations might be most beneficial to students. We believe that by doing these things the preservice teachers were developing MKTF. We now turn to another intriguing element of the preservice teachers' engagement with the activities, specifically how they made sense of their own mathematical understandings through analyzing the language used by the children.

Expressing mathematical understanding through language

Another important component of MKT is the choice of words that teachers use to express mathematical concepts as this language provides insight into their mathematical understandings. Throughout the module activities, we witnessed several instances of preservice teachers expressing their mathematical understanding of fractions through their discussion of what the children were doing. During the analysis of the children's written work, statements such as:

1. "They realize that it [numerator] is part of a whole and think in terms of area."
2. "Student Y didn't understand how to do improper fractions."

were overheard in the preservice teachers' conversations as well as seen in their written responses. Some of the preservice teachers' written statements during the investigation of the children's verbal explanations were:

3. "She sees 3 from 3/8 and only 1 from ½ so she concluded 3/8 was bigger. She clearly understands how to compare fractions or sees the shading as 3/8 and the ½ as maybe 4/8."
4. "He understands the relationship btw the num. & deno. & that he must show each fraction w/ the same unit."

The language used by the preservice teachers in the statements above highlight the active involvement it takes both physically and mentally for the participants to make connections from their ordinary language to mathematical language (Pirie, 1998). Word choices such as *whole, bigger, area,* and *relationship* have a specific contextual meaning that allows participants to make sense of the classroom activities and make links between the children's language and the symbolic language of mathematics. Using these terms as a means to evaluate what the children are thinking shows that the preservice teachers have a specific mathematical understanding of fractions, and that the language used by the child is the tool by which they connect to their own knowledge base. For example, in statement 3 a preservice teacher discussed with her classmates that a child understood fractions by comparing numerators.

It seems that when the child explained how she compared fractions it triggered recognition in the preservice teacher's own mathematical register. Specifically, the preservice teacher was able to utilize the child's language to reflect on a familiar way of comparing fractions (i.e. getting a common denominator and comparing numerators). We contend that our module provided a space that allowed preservice teachers to verbalize the children's fractional understanding, as well as their own, by discussing the language the children used, and that by doing so, the preservice teachers are acquiring components of MKTF.

As an aside, it is important to note the confident language used by the preservice teachers throughout their statements, e.g., phrases and terms such as *clearly understands* and *realize.* The use of such language points to an important aspect when utilizing children's thinking with preservice teachers, namely that one must be careful not to declare knowledge of what someone is thinking based on small segments that are taken out of context. Generally preservice teachers have very little teaching experience. Because of this, they might be prone to think in a very "black and white" fashion, expressing sentiments such as "if s/he would have just done (blank), then s/he would have understood." The preservice teachers' responses during some of our module activities underscore the necessity of teacher educators to present preservice teachers with multiple scholarly experiences regarding elementary children's thinking that challenge this “black and white” mentality. If preservice teachers are challenged in this way, hopefully they will start to realize that understanding is complex and not easily diagnosed.

Module reflection

Reflecting back, the preservice teachers seemed extremely engaged in both analyzing the children’s written work and examining the verbal explanations. However, we feel that the mathematical concepts the preservice teachers were learning were not prominent enough in the activities. In retrospect, the module activities were too evaluative, in terms of correcting children’s mathematical mistakes through teaching interventions, and not exploratory enough to allow the preservice teachers to examine at a deeper level the content inherent in the children’s work. Therefore, we would like to modify the activities so they move away from a teaching-centered focus to more of a foundational mathematics focus. This would involve developing a clearer connection between the fractional concepts and analyzing the children’s thinking.

Another related issue was that the preservice teachers struggled with how to discuss mathematical content. This fact does not surprise us since addressing mathematics content through the examination of children’s work was a new experience for all participants, and because focusing on pedagogy is probably more interesting to the preservice teachers given their chosen careers. We do believe that utilizing group work and video technology provided preservice teachers opportunities for rich interactions around course content that may otherwise be limited within a traditional lecture format. However, in future implementations of the module, we would like time set aside in the beginning for the course instructor to discuss with the preservice teachers the goals of the module and to have these goals revisited throughout the implementation of the activities. We hope that this, coupled with the clearer focus on content, will make the conversations more content rich.

Questions for future research

Several questions naturally arose during the implementation of the activities which we feel merit further investigation. First, what was it about the children's written work and explanations that promoted the conversations that occurred? Was it the framing of the activities by the course instructor, or was it the dynamics of this particular group of preservice teachers? Observing a classroom of predominantly Anglo females in their early twenties analyze and make generalizations about students' fractional number understandings was intriguing. How would the discussions have been enriched if the classroom dynamics were more heterogeneous? Finally, what specific mathematical content did the preservice teachers learn from participating in the activities, and were the mathematical understandings about fractions achieved in this setting any different than those of the students who did not engage in the activities?

Closing remarks

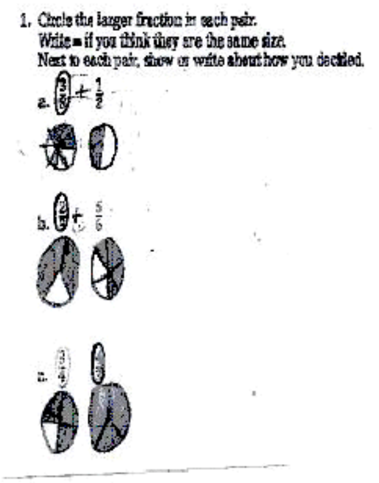
As the field of mathematics education pushes forward to develop the knowledge base of preservice teachers, incorporating field-based materials is important for their development of MKT. Through the development and implementation of our module activities, we see the value of developing instructional materials around children's written work and verbal explanations. We contend that the preservice teachers engaged in our module activities were able to develop their MKTF by both reflecting on what it means to represent a fraction and by discussing their thoughts about what they were seeing in the children's work and hearing in the children's explanations. This use of the children's work and explanations also allowed the preservice teachers the chance to "interact" with children in a course where there is no field-based practicum. Since they are seeing and hearing how children are thinking about mathematics, using these types of activities provides a lot of preservice teachers motivation to understand the mathematics concepts more deeply.

It is important to note that this article provides just a brief glimpse into what we believe is an important way to help preservice teachers acquire the mathematical knowledge needed for teaching, namely using activities centered on children's thinking. However, the use of materials such as our module in a mathematics for teaching course is not common, and there is no prescribed structure for implementation. Therefore, we call upon the field to develop and use these types of materials in order to help other teacher educators better organize and facilitate similar activities in their own courses. Showing children explaining their thinking not only helps preservice teachers develop flexibility and proficiency in understanding children's cognitive processes (Tirosh 2000), but also deepens their own mathematical understandings.

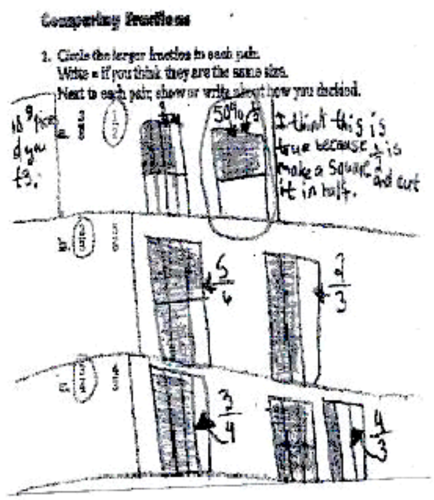
Figure 1: A unit developed around Student Y's and Student D's written work and Student Y's videotaped explanation.

SECTION I: Compare student Y's and student D's written work, specifically looking at problem (a).

Student Y's work:



Student D's work:



(i). Based on the work you see, what concept(s) related to fractions can you hypothesize that both students Y and D had a deep understanding of?

(ii). Student Y and student D came up with different answers. Based on the work you see, theorize some reasons for the different answers.

SECTION II: In the following video clip, Student Y explains how she compared $\frac{3}{8}$ and $\frac{1}{2}$.

- Briefly describe the student's method.
- What mathematical understandings does the student appear to have?
- Think back to problem 1 part 2 of the "Analyzing Student Work" worksheet. How do your thoughts from looking at the student's worksheet compare to her explanation?

Figure 2: One preservice teacher's representation of the fraction 5/6

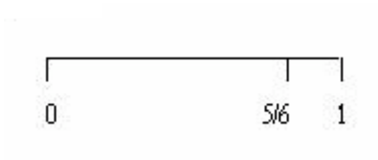
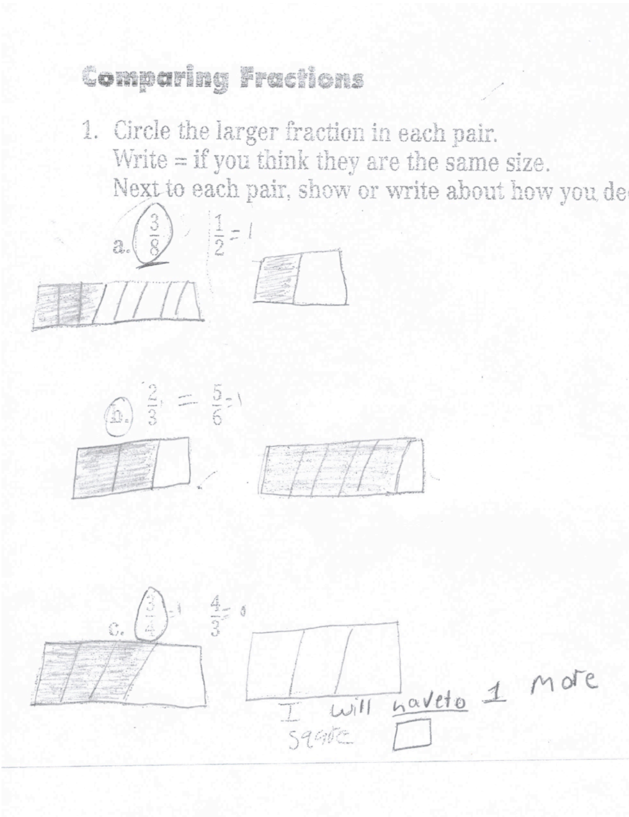


Figure 3: Student T's written work and explanation of part b.



Key: F: Facilitator; T: Student T;
OCI: Off-Camera Individual

1. T: I think that they're equal because 2 take away 3 equals - you have 1 left and 5 take away 6, I mean 6 take away 5, you still have 5 left on each one. So I thought they were both equal.

2. F: So you have one piece left on each one of your drawings, don't you? ((F gestures towards T's drawing)) And so you think they're equal because you have one left on each one? Do think those pieces that are left are the same size? [T: Mmm-hmm.] Yeah? You think they're the same size?

3. T looks at his worksheet and thinks for about 25 seconds. Teachers whispering to each other.

4. OCI: What did you say about 'b' again? Can you explain it again? You're looking at how many pieces are left?

5. T: I thought it was equal because there's both 1 left on each one ((gesturing towards the two un-shaded portions of his drawing)), and I thought it was equal.

6. OCI 2: What do you mean you thought they were equal?

7. T: On this one ((gesturing towards the right side of his drawing)) 2, you have 3, and you take away 2 out of that group and you have one left. Then you have 6 take away 5, you have one left, and they're equal 'cause you both have one left for each one.

References

Ball, Deborah. "The Mathematical Understandings that Prospective Teachers Bring to Teacher Education." *The Elementary School Journal* 90, no. 4 (1990): 449-466.

Hill, Heather, Stephen Schilling, and Deborah Ball. "Developing Measures of Teachers' Mathematics Knowledge for Teaching." *The Elementary School Journal* 105, no. 1 (2004): 11-30.

Pirie, Susan. "Crossing the Gulf between Thought and Symbol: Language as (Slippery) Stepping Stones." In *Language and Communication in the Mathematics Classroom*, edited by Heinz Steinbring, Maria Bartolini Bussi, and Anna Sierpinska, pp. 7-29. Reston, VA: National Council of Teachers of Mathematics, 1998.

Timmerman, Maria. "The Influences of Three Interventions on Prospective Elementary Teachers' Beliefs about the Knowledge Base Needed for Teaching Mathematics." *School Science and Mathematics* 105, no. 4 (2004): 369-382.

Tirosh, Dina. "Enhancing Prospective Teachers' Knowledge of Children's Conceptions: The Case of Division by Fractions." *Journal for Research in Mathematics Education* 31, no. 1 (2000): 5-25.